115 Week 1 Notes

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Claim: If $A_1, \ldots, A_n, B$ are sets then $B \cap (A_1 \cup \cdots \cup A_n) = (B \cap A_1) \cup \cdots \cup (B \cap A_n)$
Proof:
Let $x \in B \cap (A_1 \cup \cdots \cup A_n)$. Then $x \in B$ and $x \in A_1 \cup \cdots \cup A_n$ so $x \in A_i$ for some i.
Hence $x \in B \cap A_i$ and $B \cap A_i \subseteq (B \cap A_1) \cup \cdots \cup (B \cap A_n)$.
Then $x \in (B \cap A_1) \cup \cdots \cup (B \cap A_n)$, and as x was arbitrary, we have that

$$B \cap (A_1 \cup \cdots \cup A_n) = (B \cap A_1) \cup \cdots \cup (B \cap A_n).$$

Now let $x \in (B \cap A_1) \cup \cdots \cup (B \cap A_n)$. Then $x \in B \cap A_i$ for some A_i.
Then $x \in B$ and $x \in A_i$. As $A_i \subseteq A_1 \cup \cdots \cup A_n, x \in A_1 \cup \cdots \cup A_n$.
Hence $x \in B \cap (A_1 \cup \cdots \cup A_n)$ and as x was arbitrary we have that

$$B \cap (A_1 \cup \cdots \cup A_n) \supseteq (B \cap A_1) \cup \cdots \cup (B \cap A_n).$$

Hence $B \cap (A_1 \cup \cdots \cup A_n) = (B \cap A_1) \cup \cdots \cup (B \cap A_n)$

Claim: If $A_1, \ldots, A_n, B$ are sets then $B \cup (A_1 \cap \cdots \cup A_n) = (B \cup A_1) \cap \cdots \cap (B \cup A_n)$
Proof:
Let $x \in B \cup (A_1 \cap \cdots \cup A_n)$. Then $x \in B$ or $x \in A_1 \cap \cdots \cup A_n$. We break the problem into two cases:
Case 1: If $x \in B$, then $x \in B \cup A_i$ for each i. Hence $x \in (B \cup A_1) \cap \cdots \cap (B \cup A_n)$.
Case 2: If $x \in A_1 \cap \cdots \cup A_n$ then $x \in A_i$ for each i. As $A_i \subseteq B \cup A_i$, $x \in B \cup A_i$.
Hence $x \in (B \cup A_1) \cap \cdots \cap (B \cup A_1)$, so $B \cup (A_1 \cap \cdots \cup A_n) \subseteq (B \cup A_1) \cap \cdots \cap (B \cup A_n)$.

Now let $x \in (B \cup A_1) \cap \cdots \cap (B \cup A_n)$. Then $x \in B \cup A_i$ for each A_i. We consider two cases: $x \in B$ and $x \notin B$.

Case 1: If $x \in B$ then $x \in B \cup (A_1 \cap \cdots \cup A_n)$.
Case 2: If $x \notin B$ then as $x \in B \cup A_i$ for each i, $x \in A_i$ for each i. Hence $x \in A_1 \cap \cdots \cap A_n$, so $x \in B \cup (A_1 \cap \cdots \cup A_n)$
Then $B \cup (A_1 \cap \cdots \cup A_n) \supseteq (B \cup A_1) \cap \cdots \cap (B \cup A_n)$.

Hence $B \cup (A_1 \cap \cdots \cup A_n) = (B \cup A_1) \cap \cdots \cap (B \cup A_n)$

Claim: If A and B are sets, then $A \setminus (A \setminus B) = A \cap B$.
Proof:
We proceed carefully by the definition of "\"

$$\begin{aligned}
A \setminus (A \setminus B) &= \{x : x \in A \text{ and } x \notin A \setminus B\} \\
&= \{x : x \in A \text{ and not}(x \in A \setminus B)\} \\
&= \{x : x \in A \text{ and not}(x \in A \text{ and } x \notin B)\} \\
&= \{x : x \in A \text{ and } (x \notin A \text{ or } x \in B)\} \\
&= \{x : x \in A \text{ and } (x \notin A \text{ or } x \in B)\} \\
&= \{x : (x \in A \text{ and } x \notin A) \text{ or } (x \in A \text{ and } x \in B)\}
\end{aligned}$$

As $(x \in A \text{ and } x \notin A)$ is never true, we must have this is equal to

$$\{x : x \in A \text{ and } x \in B\} = A \cap B$$